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A METHOD FOR DETERMINING THE AERODYNAMIC CHARACTERISTICS OF TWO- AND THREE-DIMENSIONAL SHAPES AT HYPERSONIC SPEEDS

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SUMMARY

A method is developed for calculating the pressures on aerodynamic shapes at very high supersonic speeds in dense air with the ratio of specific heats equal to 1. The method is applicable to any body of revolution at zero angle of attack and to any two-dimensional profile. The results of the present paper are compared with previous work of von Kármán and Epstein on this problem and the differences explained.

Some aerodynamic characteristics of several shapes are calculated, and the lift and drag coefficients are shown to be dependent upon the thickness ratio, thickness distribution, and angle of attack.

INTRODUCTION

The possibility of constructing airplanes and missiles capable of traveling at extremely high supersonic speeds is of ever increasing importance. Reference 1, for instance, shows that Mach numbers of the order of 30 or 40 are necessary for obtaining very long ranges with rockets. The largest part of such flight paths will probably be at very high altitudes. The aerodynamics of flight at these altitudes has been treated by Sänger and Tsien in references 2 and 3. The part of the flight in dense air (that is, in a fluid which may be considered a continuum) is also of importance since the surface temperatures and air loads may reach very high values. The problem of aerodynamic heating has been investigated in reference 4. It is the purpose of the present paper to investigate the air loads encountered.

Busemann has done much of the original work in developing a theory for hypersonic flows, however his work on this subject is not generally known in this country. After the present paper had been completed, Dr. Busemann made available to the authors a copy of his original work (reference 5). The method developed in the present paper is in substantial agreement with that reference. Von Kármán, in his Volta Congress address of 1935

(reference 6), referred to Busemann's theory but in applying the method imposed physically impossible conditions on the flow. Another theory has been presented by Epstein (reference 7); however, since he neglected the pressure relief due to centrifugal force, his method is applicable only to wedges and cones.

SYMBOLS

C_D	drag coefficient based on unit chord or maximum cross section
C_L	lift coefficient based on unit chord
D	drag
F	fineness ratio $\left(\frac{\text{Distance from nose to maximum cross section}}{\text{Maximum thickness}}\right)$
f	function
L	lift
m	mass
M	Mach number
P	pressure coefficient
p	static pressure
q	dynamic pressure
r	radius of body of revolution
R	radius of curvature
t	thickness ratio
U	free-stream air velocity
x,y	coordinate axes
α	angle of attack
β	angle between surface and free-stream direction
γ	ratio of specific heats
ρ	density

θ	shock-wave angle
ϕ	angle between surface and x-axis

Subscripts:

o	free stream; due to form
a,b	ahead of and behind shock wave
1,2	points of zero pressure coefficient
c	due to centrifugal force
i	end point of any surface element
l	lower surface
L	due to lift
max	maximum
N	normal
s	due to shock
T	tangential; total due to pressure
u	upper surface

METHOD OF ANALYSIS

In order to calculate the pressures on bodies at hypersonic speeds it is necessary to determine the position of the nose shock wave. First consider two equations from the theory of shock waves. The relation between the density ratio across a shock wave and the Mach number normal to the shock is

$$\frac{\rho_b}{\rho_a} = \frac{\gamma + 1}{\frac{2}{M_N^2} + \gamma - 1}$$

the relation between density ratio and the shock-wave angle θ and deflection angle β is

$$\frac{\rho_b}{\rho_a} = \frac{\tan \theta}{\tan (\theta - \beta)}$$

If M_N is very large $\frac{2}{{M_N}^2}$ can be neglected and the following very useful approximation for high Mach number flow is obtained:

$$\frac{\rho_b}{\rho_a} \approx \frac{\gamma + 1}{\gamma - 1}$$

For $\gamma = 1$

$$\frac{\rho_b}{\rho_a} \longrightarrow \infty$$

and for $\gamma = 1.4$

$$\frac{\rho_b}{\rho_a} \longrightarrow 6$$

Since θ is not generally equal to 90° ($\tan \theta \neq \infty$), for $\gamma = 1$

$$\theta = \beta \tag{1a}$$

and for $\gamma = 1.4$ and slender bodies

$$\theta = 1.2\beta \tag{1b}$$

These relations may be applied only to combinations of free-stream Mach numbers and surface angles which result in very large Mach numbers normal to the shock (surface). At the limit $M = \infty$ all surface angles may be considered.

The shock equations have been developed on the assumption of an irreversible adiabatic process (no heat transfer except that occurring within the shock wave itself). Epstein (reference 7) points out that the compression at very high Mach numbers results in extremely high temperatures which in turn cause large heat losses due to radiation and conduction. These heat losses limit the temperature rise to a value much lower than that determined by the adiabatic law. A better approximation to the shock equations at hypersonic speeds may possibly be obtained by use of the assumption that $\gamma = 1$ in the preceding equations. Equations (1) show that the results are not critically affected by the value chosen for γ; hence, for simplicity, the calculations in this paper will be based on the relation $\gamma = 1$.

Equations (1) indicate that at very high supersonic Mach numbers the shock wave follows the surface. Since the flow is supersonic the air ahead of the shock wave is not affected by the body, and hence the

field of disturbed flow around the body is limited to a thin layer behind the shock which has been called the hypersonic boundary layer. The velocity in this layer may be determined from the shock equations.

From the following figure it is apparent that the mass flow (per unit area) through the shock wave is

$$\rho_a U_{a_N} = \rho_b U_{b_N} \tag{2}$$

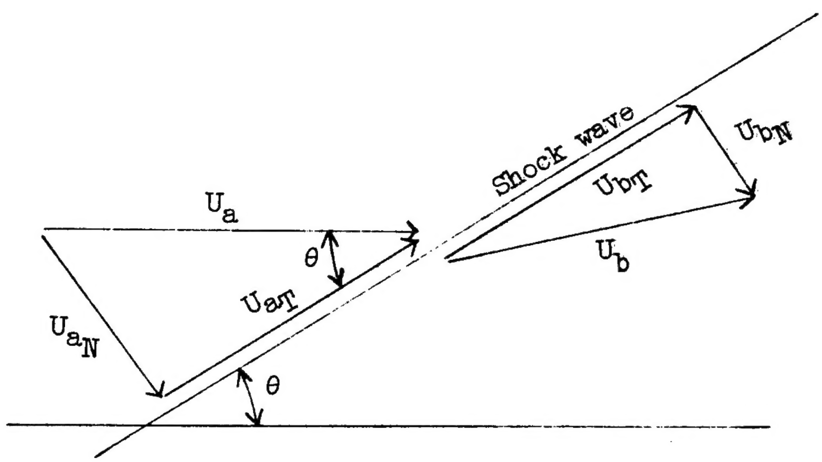

Figure 1.– Velocities near a shock wave.

Since $\frac{\rho_a}{\rho_b} \longrightarrow 0$ it follows that

$$\frac{U_{b_N}}{U_{a_N}} \longrightarrow 0$$

Therefore U_{b_N} will be considered negligible relative to U_{a_N}.

Equating momentum before and after the shock in a tangential direction gives

$$\left(\rho_a U_{a_N}\right) U_{a_T} = \left(\rho_b U_{b_N}\right) U_{b_T} \tag{3}$$

As determined from figure 1, equations (1), (2), and (3) the tangential velocity behind the shock (along the surface) is

$$U_{b_T} = U_{a_T} = U_a \cos\theta = U_a \cos\beta$$

At very high Mach numbers with $\gamma = 1$ the region of disturbed flow is confined to an infinitesimal layer between the surface and the shock wave. Since the cross section of this disturbed layer is very small and the density is very large, the acceleration in the direction of flow is negligible. Hence the assumption is made that the speed of a given mass of the fluid remains constant along the surface at the value $U_o \cos \beta$. This assumption is the same as that made by Busemann and von Kármán. A sketch of the velocities over a surface is given in figure 2.

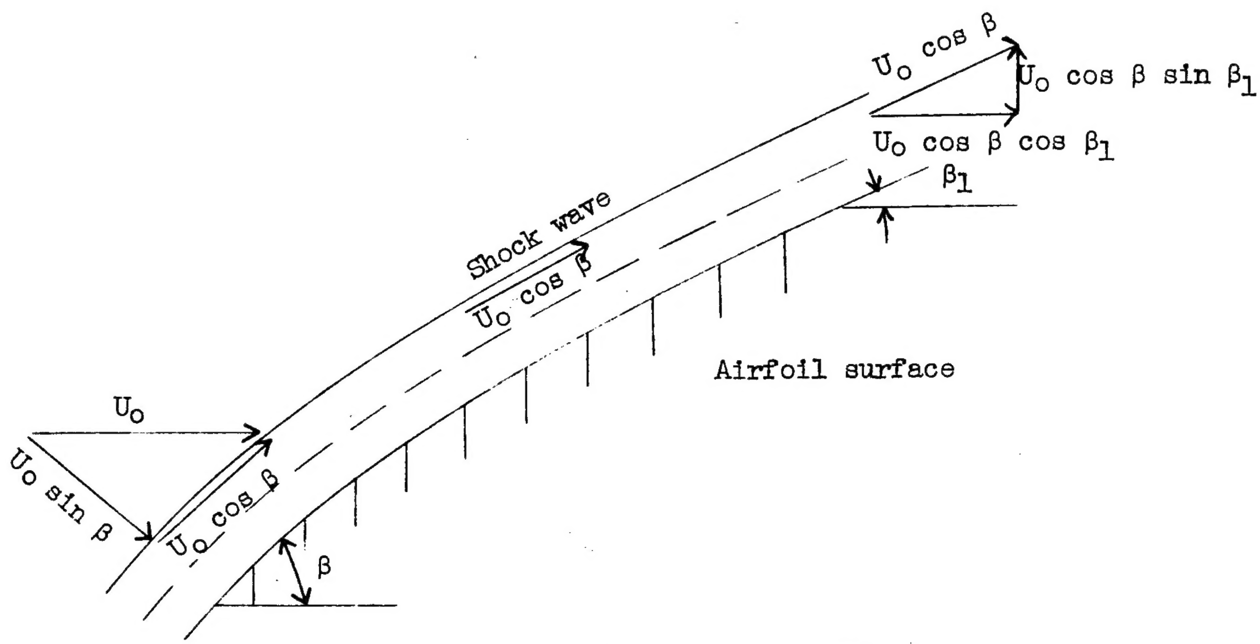

Figure 2.– Velocities over a surface.

The drag of the body is found from the rate of change of the momentum of the air in the stream direction. The air mass per unit time entering an elemental area of shock for a body of revolution at zero angle of attack is

$$2\pi r \rho_o U_o \, dr$$

and the change in velocity in the stream direction from the time the elemental mass enters the shock at β until it leaves the surface being considered at β_1 is

$$U_o - U_o \cos \beta \cos \beta_1$$

The drag is then

$$D = \int_0^{r_1} 2\pi r \rho_o U_o (U_o - U_o \cos \beta \cos \beta_1) \, dr \tag{4}$$

where $\cos \beta$ is a known function of r, and β_1 corresponds to the surface angle at the point r_1 where the flow diverges from the surface. The location of r_1 will be determined later in the analysis. Equation (4) differs from that given by von Kármán inasmuch as he assumes that the flow leaves the surface in the free-stream direction $(\beta_1 = 0, r_1 = r_{max})$. This assumption would require surface pressures less than absolute zero and is therefore not physically justified. For a two-dimensional body the elemental mass is $\rho_o U_o \, dy$ and the drag for one surface becomes

$$D = \int_0^{y_1} \rho_o U_o^2 (1 - \cos \beta \cos \beta_1) \, dy \tag{5}$$

For a cone β remains constant at the same value as β_1. Then the drag of a cone at zero angle of attack simplifies to

$$D = 2\pi \int_0^{r_{max}} \rho_o r U_o^2 (1 - \cos^2 \beta_1) \, dr$$

$$= \pi r_{max}^2 \rho_o U_o^2 \sin^2 \beta_1 \tag{6}$$

or

$$C_D = 2 \sin^2 \beta_1 \tag{7}$$

Equation (6) is identical with Epstein's result (reference 7). Since Epstein neglected the effect of the pressure drop due to surface curvature, his method would be expected to be in agreement with the present work only for straight-side bodies.

The pressure coefficient behind a shock wave is frequently given as

$$P = \frac{2 \sin \beta \sin \theta}{\cos (\theta - \beta)} \tag{8}$$

Equations (1) and (8) combine to give the pressure coefficient behind a shock at hypersonic speeds

$$P = 2 \sin^2 \beta \tag{9}$$

A comparison of equations (7) and (9) indicates that, for $\gamma = 1$, two- or three-dimensional bodies that have no surface curvature in the stream direction have the same pressure on the surface as behind the leading shock. When the flow is curved in the stream direction, the difference in pressure from the shock wave to the surface equals the centrifugal force due to the curvature of the flow. The pressure coefficient behind the shock depends on the surface slope, and the pressure relief due to curvature depends on the local air mass, velocity, and radius of curvature. The maximum curvature that the flow can have for a given shock pressure, mass, and velocity occurs when the pressure on the airfoil side of the flow drops to zero. If a pressure still lower is needed to turn the flow sufficiently to follow the airfoil or body surface, it is physically impossible for the flow to follow the surface and it effectively "separates." For the very high dynamic pressures encountered at hypersonic speeds the pressure coefficient P corresponding to zero pressure can be considered as zero. It follows that the limits of integration r_1 and y_1 of equations (4) and (5) correspond to the points where the surface pressure coefficients go to zero. Thus, in order to determine the limits of integration for the drag integral it is necessary to know the local pressure coefficient.

If in equation (5) the subscripts 1 are replaced by i where the subscript i corresponds to the end point of any element, the expression will give the drag for the part of the body ahead of the point (x_i,y_i)

$$D_i = \rho_o U_o^2 \int_o^{y_i} (1 - \cos \beta \cos \beta_i)\, dy$$

In terms of drag coefficient based on unit chord

$$C_{D_i} = 2 \int_o^{y_i} (1 - \cos \beta \cos \beta_i)\, dy \tag{10}$$

The surface pressure coefficient at the point (x_i,y_i) is then

$$P_i = \frac{dC_{D_i}}{dy_i}$$

For $x_i \leqq x_1$

$$P_i = 2 \sin^2\beta_i + 2 \sin \beta_i \frac{d\beta_i}{dy_i} \int_o^{y_i} \cos \beta\, dy \tag{11a}$$

and for $x_i \geqq x_1$

$$P_i = 0 \tag{11b}$$

Similarly, for a three-dimensional body of revolution the elemental drag coefficient based on the maximum cross section of the complete body is

$$C_{D_i} = 16F^2 \int_0^{r_i} r(1 - \cos \beta \cos \beta_i)\, dr$$

and on the surface

$$P_i = \frac{1}{8F^2 r_i} \frac{dC_{D_i}}{dr_i}$$

For $x_i \leqq x_1$

$$P_i = 2 \sin^2\beta_i + \frac{2 \sin \beta_i}{r_i} \frac{d\beta_i}{dr_i} \int_0^{r_i} r \cos \beta \, dr \tag{12a}$$

and for $x_i \geqq x_1$

$$P_i = 0 \tag{12b}$$

where the fineness ratio $F = \dfrac{\text{Length from nose to maximum cross section}}{\text{Maximum thickness}}$

The value of $y_i = y_1$ (or $r_i = r_1$) is determined by equating the pressure coefficient to zero. This is the proper value to be used in evaluating the total drag integral. (See equations (4) and (5).) The flow separates from the surface at the point (x_1,y_1) and is bounded by a zero-pressure streamline. Any body within this zero-pressure region contributes no drag.

In equations (11a) and (12a) the first term is the pressure coefficient behind the shock and the second term is the pressure coefficient due to centrifugal force.

A good approximation to the surface pressure coefficient for slender profiles may be obtained in a simple manner. Let $y = f(x)$ be the shape of the body or airfoil then

$$\frac{dy}{dx} = f'(x)$$

and

$$\frac{d^2y}{dx^2} = f''(x)$$

The pressure coefficient behind the shock (equation (9)) becomes

$$P_s = \frac{2f'^2}{1 + f'^2}$$

If the velocity is not changed appreciably by the shock (true only for slender bodies or airfoils), the pressure drop due to centrifugal force is

$$(\Delta p)_c = \frac{mU_o{}^2}{R} = \frac{2qy}{R}$$

or

$$P_c = 2\,\frac{y}{R}$$

where the radius of curvature

$$R = \frac{(1 + f'^2)^{3/2}}{f''}$$

The pressure on the surface is the shock pressure plus the change due to centrifugal force:

$$P = P_s + P_c$$

$$= \frac{2f'^2}{1 + f'^2} + \frac{2ff''}{(1 + f'^2)^{3/2}}$$

Since f'^2 is small compared with 1, the pressure coefficient on a slender airfoil may be taken as

$$P = 2(f'^2 + ff'') \tag{13}$$

For a three-dimensional body the air mass over each surface element is

$$\frac{\rho_o U_o \pi r^2}{2\pi r U_o} = \frac{\rho_o r}{2}$$

By similar reasoning the pressure coefficient for slender, three-dimensional bodies may be approximated as

$$P = 2f'^2 + ff'' \tag{14}$$

The lift of a body or airfoil results from the downward momentum imparted to the air stream. The same considerations used in determining the drag from the horizontal momentum apply to the calculation of lift from the vertical momentum. The lift for the upper surface of a two-dimensional airfoil is then of the form

$$L = -\rho_o U_o^2 \sin \beta_1 \int_o^{y_1} \cos \beta \, dy \tag{15}$$

where y_1 is the point where the surface pressure becomes negligible.

APPLICATION OF THEORY

Lift and drag of a flat plate.- Equation (9) can be used as the basis for calculating the lift of straight-surface airfoils. The lift coefficient of a flat plate at a small angle of attack can be written as

$$C_L = P_l - P_u$$

Since the suction pressures are negligible in comparison with the pressure rises

$$C_L = 2\alpha^2 - 0 = 2\alpha^2$$

$$\frac{dC_L}{d\alpha} = 4\alpha$$

and

$$C_D = C_L\alpha = 2\alpha^3$$

Lift and drag of wedge airfoil.– The lift of a single-wedge airfoil is easily obtained by the same method used in the preceding section. For angles of attack α less than the semi-nose angle

$$C_L = P_l - P_u$$

$$C_L = 2\left[(\beta + \alpha)^2 - (\beta - \alpha)^2\right]$$

$$C_L = 8\beta\alpha = 4t\alpha \tag{16}$$

and

$$\frac{dC_L}{d\alpha} = 8\beta = 4t$$

The drag coefficient of a single-wedge airfoil at an angle of attack α which is less than the semi-nose angle is

$$C_{D_T} = 2\left[(\beta + \alpha)^3 + (\beta - \alpha)^3\right]$$

$$= 4\beta^3 + 12\beta\alpha^2$$

This can be broken into a form drag C_{D_o} and a drag due to lift C_{D_L}:

$$C_{D_o} = 4\beta^3 = \frac{t^3}{2}$$

$$C_{D_L} = 12\beta\alpha^2 \tag{17}$$

The combination of equations (16) and (17) gives the following expression for drag due to lift for this airfoil:

$$C_{D_L} = \frac{3}{2} C_L\alpha = \frac{3}{8} \frac{C_L{}^2}{t}$$

For angles of attack larger than β, the lift and drag of the wedge and flat plate are the same for the same slopes of the lower surfaces.

Lift, drag, and pressure on a two-dimensional parabolic-arc airfoil.- From equation (5) the drag coefficient for a symmetrical two-dimensional airfoil of unit chord may be expressed as

$$C_D = 4\int_0^{y_1} (1 - \cos\beta \cos\beta_1)\, dy$$

For the parabolic airfoil defined by $y = \pm tx\left(1 - \frac{x}{2}\right)$ where t is the thickness ratio, the drag coefficient becomes

$$C_D = 2tx_1(2 - x_1) + \frac{4}{t}\left[1 - \frac{\sqrt{1 + t^2}}{\sqrt{1 + t^2(1 - x_1)^2}}\right] \tag{18}$$

The value of x_1 for which the pressure coefficient is zero is found to be

$$x_1 = 1 - \frac{1}{t}\sqrt{(1 + t^2)^{1/3} - 1}$$

Substitution of x_1 in equation (18) gives the following equation for drag coefficient:

$$C_D = 2t - \frac{6}{t}\left[(1 + t^2)^{1/3} - 1\right]$$

or for simplicity of numerical calculations it may be expressed in a series as

$$C_D = \frac{2t^3}{3}\left(1 - \frac{5}{9}t^2 + \ldots\right)$$

The pressure coefficient on the surface of the airfoil at any point (x_i, y_i) is found from equations (11). Then, for $x_i \leqq x_1$,

$$P = 2\left\{1 - \frac{\sqrt{1 + t^2}}{\left[1 + t^2(1 - x_i)^2\right]^{3/2}}\right\}$$

and for $x_i \geqq x_1$

$$P = 0$$

or, approximately,

$$P = t^2\left[3(1 - x_i)^2 - 1\right] + \frac{t^4}{4}\left[1 + 6\ (1 + x_i)^2 - 15(1 - x_i)^4\right] + \ldots$$

The pressure coefficient found by the approximate method (equation (13)) is, for $x_i \leqq x_1$,

$$P = t^2\left[3(1 - x_i)^2 - 1\right]$$

The agreement of the pressure coefficients found by these two methods is very good as can be seen in figure 3(a) where the pressure coefficients on the surface as well as the pressure behind the shock are given for a parabolic-arc airfoil of thickness ratio equal to 0.10.

The lift coefficient (based on a unit chord) is found from equation (15) to be

$$C_L = 2 \sin \beta_2 \int_o^{y_2} \cos \beta \, dy + 2 \sin \beta_1 \int_o^{y_1} \cos \beta \, dy$$

$$= 2 \sin \beta_2 \int_o^{x_2} \sin \beta \, dx + 2 \sin \beta_1 \int_o^{x_1} \sin \beta \, dx$$

where (x_1,y_1) and (x_2,y_2) are the points of flow separation on the upper and lower surfaces, respectively, and β is the angle between the airfoil surface and stream direction. Let $\phi = \beta - \alpha$ for the upper surface and $\phi = \beta + \alpha$ for the lower surface where ϕ is the angle between the airfoil surface and the x-axis. Then for small angles

$$\sin(\phi - \alpha) \approx \phi - \alpha$$

For the symmetrical parabolic airfoil defined by $y = tx\left(1 - \frac{x}{2}\right)$ the lift coefficient based on a unit chord is found to be

$$C_L = \left[t^2x_2^3 - 3tx_2^2(t + \alpha) + 2x_2(t + \alpha)^2\right] - \left[t^2x_1^3 - 3tx_1^2(t - \alpha) + 2x_1(t - \alpha)^2\right] \tag{19}$$

The pressure coefficients for the upper and lower surfaces are found from equations (11). Setting the pressure coefficients found from equations (11) equal to zero permits the determination of the values of x_1 and x_2

$$P_u = 3t^2x_1^2 - 6tx_1(t - \alpha) + 2(t - \alpha)^2 = 0$$

$$P_l = 3t^2x_2^2 - 6tx_2(t + \alpha) + 2(t + \alpha)^2 = 0$$

The lift is then found by substituting these values of x_1 and x_2 into equation (19). For $\alpha < t$

$$C_L = \frac{2\sqrt{3}}{9}\left[\frac{(t + \alpha)^3}{t} - \frac{(t - \alpha)^3}{t}\right]$$

or for $t \leqq \alpha \leqq t\left(\frac{1 + \sqrt{3}}{2}\right)$

$$C_L = \frac{2\sqrt{3}}{9}\left[\frac{(t + \alpha)^3}{t}\right]$$

or for $\alpha \geqq t\left(\frac{1 + \sqrt{3}}{2}\right)$

$$C_L = t^2 - 3t(t + \alpha) + 2(t + \alpha)^2$$

The pressure distribution for a ten-percent-thick parabolic airfoil is shown in figure 3(b) for $\alpha = 0.05$ radian. If α is larger than the semi-nose angle, the effect of the upper surface on the total lift is zero since suction pressures are neglected.

Drag and pressure on a parabolic body of revolution.- The drag of a parabolic body of revolution at zero angle of attack may be determined by the same method used for the two-dimensional bodies. Let the surface be described by the equation

$$r = \frac{x}{F}\left(1 - \frac{x}{2}\right)$$

where F is the fineness ratio of the body. The expression for drag coefficient is then

$$C_D = 16F^2 \int_0^{r_1} (1 - \cos \beta \cos \beta_1) r \, dr$$

which reduces to

$$C_D = 2x_1{}^2(2 - x_1)^2 + \frac{8F^4}{3}\left[\frac{3}{F^2} + 2 - \frac{(1 - x_1)^2}{F^2} - 2\,\frac{(F^2 + 1)}{F^2}\,\frac{\sqrt{\frac{F^2 + 1}{F^2}}}{\sqrt{1 + \frac{(1 - x_1)^2}{F^2}}}\right]$$

where x_1 is again the point of zero pressure coefficient.

The expression for C_D expanded in a series is

$$C_D = \frac{1}{3F^2}\left[1 + 3(1 - x_1)^2 - 9(1 - x_1)^4 + 5(1 - x_1)^6\right]$$
$$- \frac{1}{24F^4}\left[3 + 4(1 - x_1)^2 + 18(1 - x_1)^4 - 60(1 - x_1)^6 + 35(1 - x_1)^8\right] + \ldots$$

The surface pressure coefficient found from equation (12) is, for $x_i \leqq x_1$,

$$P = 2\left(1 + \frac{2F^2}{3x_i(2 - x_i)}\left\{1 - \frac{\left(1 + \frac{1}{F^2}\right)^{3/2}}{\left[1 + \frac{1}{F^2}(1 - x_i)^2\right]^{3/2}}\right\}\right)$$

and, for $x_i \geqq x_1$,

$$P = 0$$

or as a series, for $x_i \leqq x_1$,

$$P = \frac{1}{12x_i(2 - x_i)}\left\{-\frac{6}{F^2}\left[1 - 6(1 - x_i)^2 + 5(1 - x_i)^4\right]\right.$$

$$\left. + \frac{1}{F^4}\left[1 + 9(1 - x_i)^2 - 45(1 - x_i)^4 + 35(1 - x_i)^6\right] + \cdots\right\} \tag{20}$$

By use of the approximation given by equation (14) the pressure coefficient may also be expressed, for $x_i \leq x_1$, as

$$P = \frac{1}{2F^2}\left(5x_i^2 - 10x_i + 4\right) \tag{21}$$

The pressure distribution over a parabolic body of revolution of fineness ratio equal to 10 as determined by equation (20) is plotted in figure 4. The value of the pressure distribution as determined by the approximate method (equation (21)) is also plotted in figure 4 together with the pressure behind the shock.

CONCLUDING REMARKS

A method for calculating the aerodynamic characteristics of two- and three-dimensional shapes at hypersonic speeds in dense air has been developed. The results of the present paper are in substantial agreement with some earlier work of Busemann but differ from the results of von Kármán and Epstein. Von Kármán, in his application of Busemann's method, has assumed that the flow leaves the surface parallel to the free stream. This assumption would require surface pressures less than absolute zero. Epstein neglected the pressure relief due to centrifugal force and therefore his results apply only to bodies with zero curvature.

The method of the present paper has been applied to several simple shapes and compared with other calculations.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., February 5, 1948

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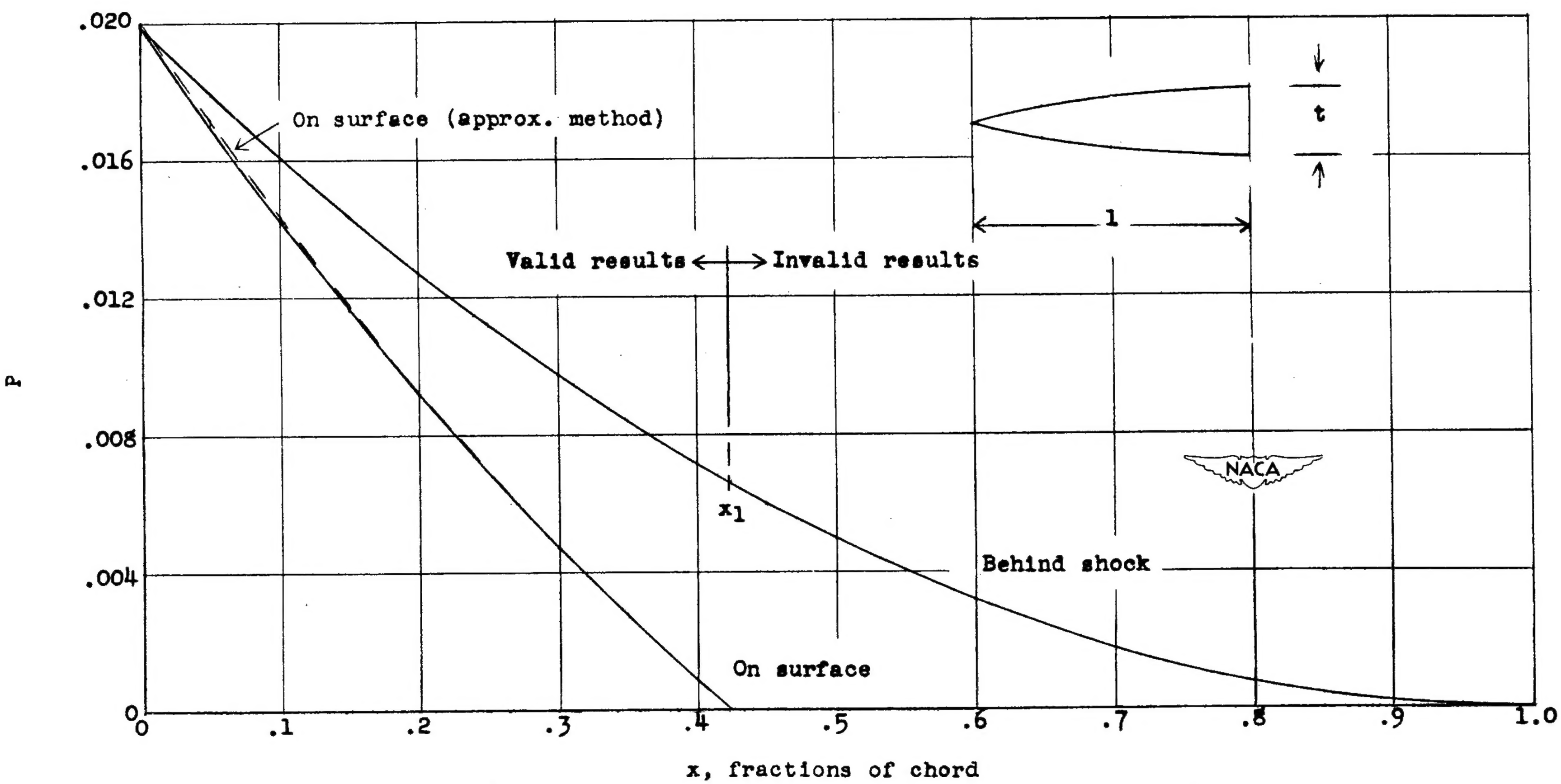
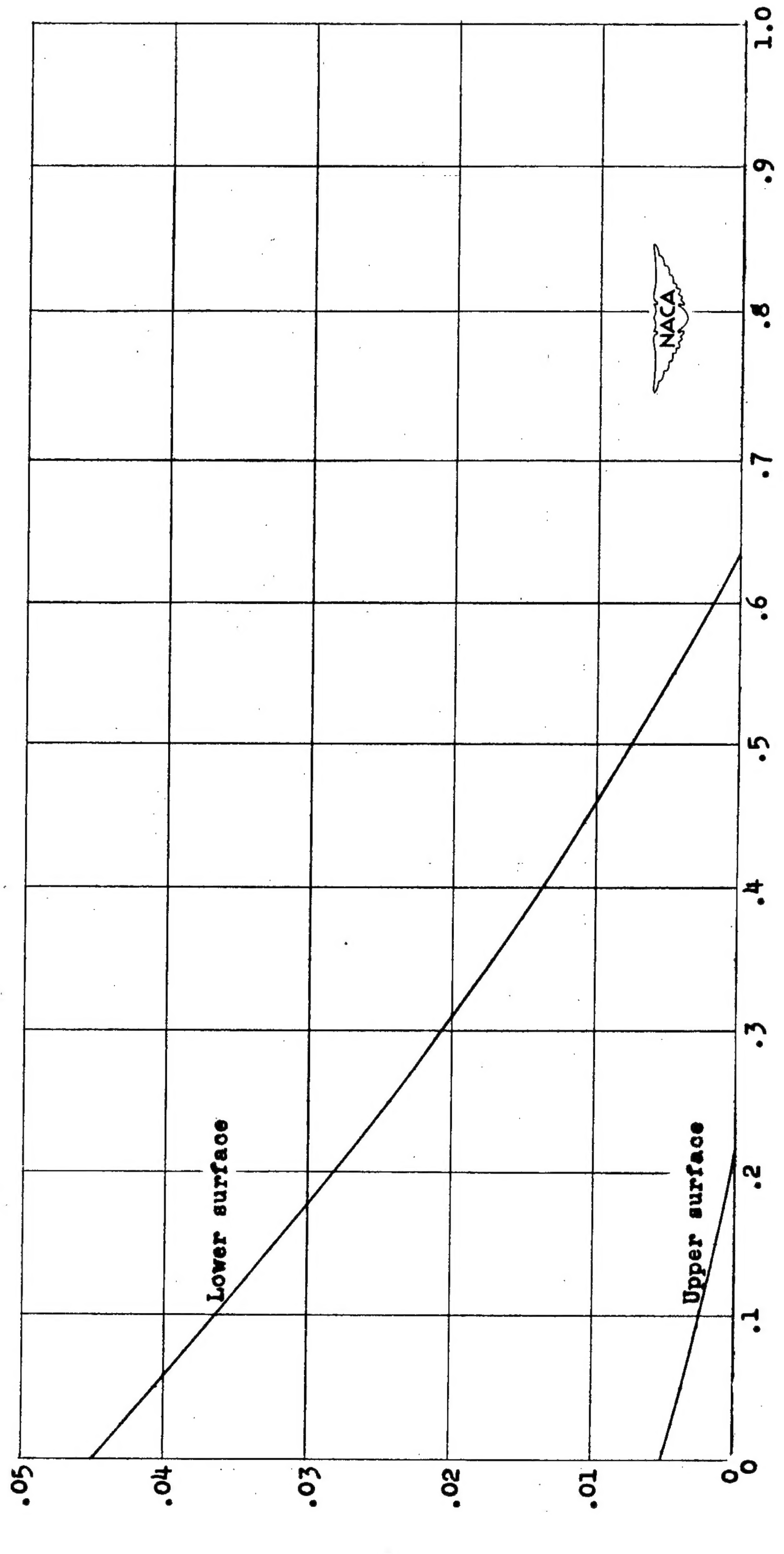


(a) α = 0 radian; C_D = 0.00066.

Figure 3.- Pressure distribution over parabolic airfoil. Thickness ratio, 0.10.



x, fractions of chord

(b) α = 0.05 radian; c_L = 0.0125.

Figure 3.- Concluded.

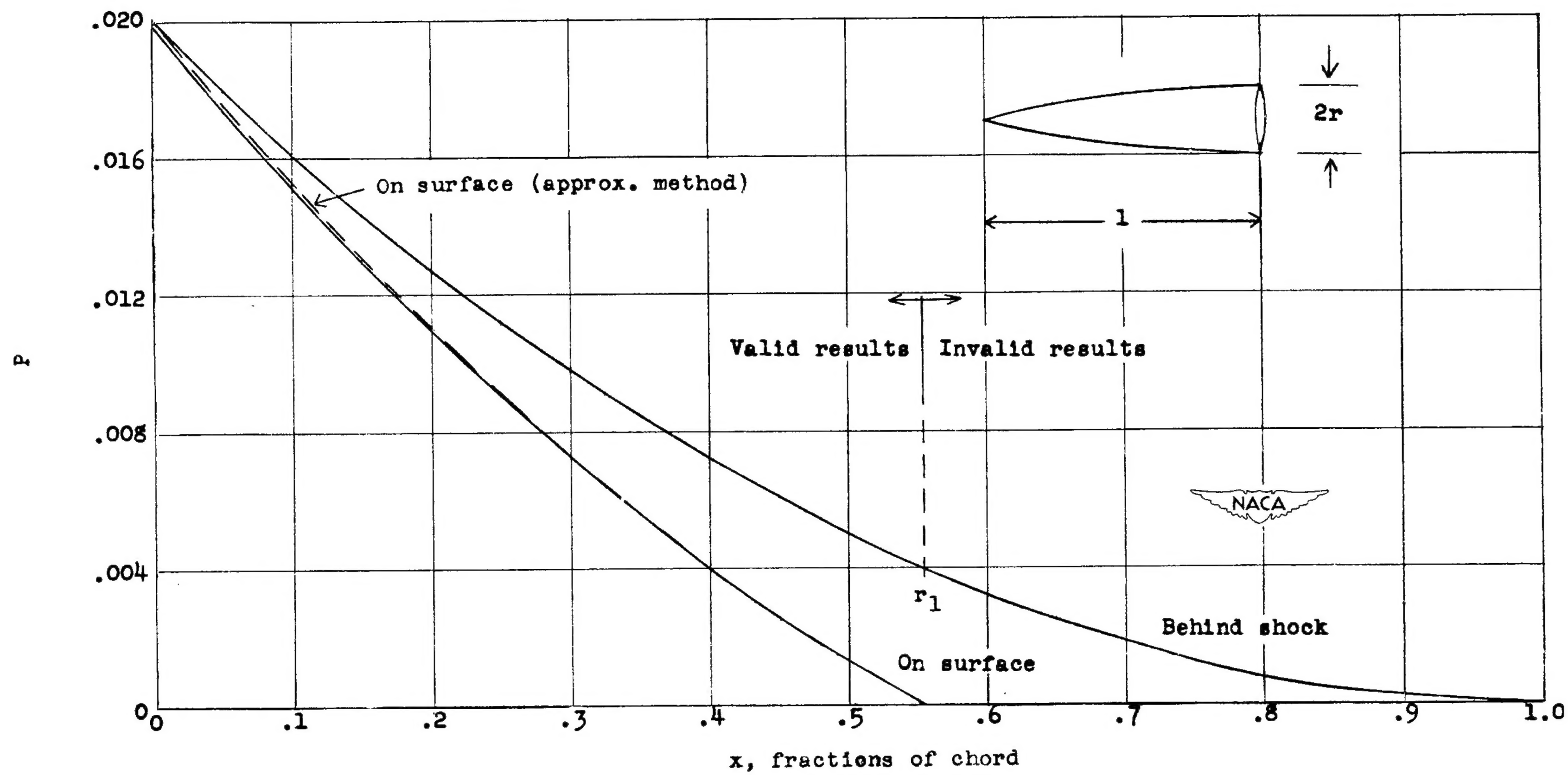


Figure 4.- Pressure distribution over parabolic body of revolution. Fineness ratio, 10; α = 0 radian; $C_D = 0.0042$.